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Formation of walls in cylindrical smectic C layers in the presence of a tilted magnetic field

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A theoretical study of the elastic properties of a smectic C liquid-crystalline system consisting of cylindrical smectic layers is presented. We show that the ground state configuration of the c-director for such a system depends on the signs of two combinations of the relevant elastic constants. With these configurations as a starting point, we describe how a set of walls under certain conditions must be formed in the system if a magnetic field is applied at an angle to the cylinder axis. An estimate of the thickness of these walls is given. We also show that, depending on whether the angle between the magnetic field and the cylinder axis is larger or smaller than the tilt of the director with respect to the smectic layer normal, the system will exhibit two qualitatively different behaviours.

1. Introduction

Smectic C liquid crystals [1] are layered structures for which the elongated molecules, being the building blocks of the system, are tilted on average with respect to the layer normal. In order to introduce the basic quantities needed to describe a S_C liquid crystal and to set the corresponding notation unambiguously we refer to figure 1. To characterize the smectic layers we introduce a unit vector **a** defining the layer normal. Assuming that the system we study is free from dislocations and of constant layer thickness, the layer normal **a** must fulfil [1] the constraint

$$\nabla \times \mathbf{a} = 0. \tag{1}$$

The average direction of the long molecular axes is defined by a unit vector **n**, the director. For each temperature in a S_C liquid crystal the angle between the director and the layer normal is fixed. This tilt angle is commonly denoted by θ. The energy required to change θ is, except very close ($T_{S_CS_A} - T < 0{\cdot}1$ K) to the transition to the smectic A phase, normally very large [2] compared to the energies associated with external fields, elastic deformations, boundary conditions etc. Thus we can in most cases take the tilt angle to be a constant, solely dependent on the temperature of the system; we only consider this case here. The projection of the director on to the smectic planes is normally described by a unit vector **c**, commonly called the c-director. To describe the direction of the c-director with respect to some reference direction within the smectic

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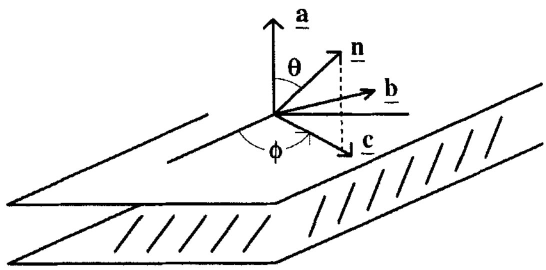

Figure 1. Notation used in the present work. The average molecular direction, the director, is given by a unit vector **n** making an angle θ with the layer normal **a**. The c-director, being a unit vector parallel to the projection of the director into the smectic planes, is denoted by **c** and is described by the angle ϕ. The unit vector **b**, which is also confined to lie within the smectic planes, is defined by the relation $\mathbf{b}=\mathbf{a}\times\mathbf{c}$.

planes we introduce the angle ϕ. In some cases, for mathematical convenience, we also find it convenient to introduce a third unit vector $\mathbf{b}=\mathbf{a}\times\mathbf{c}$.

In order to characterize the configuration of a S_C liquid crystal (assuming a constant tilt) we need to specify the spatial variations of **a** and **c**. While the only requirement imposed on **c** (disregarding the possiblity of the formation of disclinations) is that it is a continuous function in space, the layer normal has to fulfil the constraint of equation (1). This severely restricts the possible ways to arrange the smectic layers in space. The simplest possible way for the smectic layers to fill space is of course to form planar layers. More complex configurations of the smectic planes fulfilling equation (1) are arrangements where the smectic layers form concentric cylinders or spheres or parts thereof. Even more complicated possible configurations are the focal conics and the Dupin cyclides [1, 3]. Once the layers have formed one of the allowed configurations, we may find them to be stable in the sense that all possible perturbations of the layer normal **a**, compatible with the given boundary conditions, will violate the constraint $\nabla\times\mathbf{a}=0$. We have introduced [4] the term geometrically stable to describe such configurations.

It is an experimental fact [5] that an arrangement of concentric cylindrical smectic layers can be formed spontaneously in S_A as well as in S_C liquid-crystalline systems. The orientation of such cylinders will, of course, be random and, in order to fill space completely, be accompanied [5] by more complicated structures. One way to form a well-defined sample with a configuration of concentric cylindrical smectic layers would be to use a compound which posesses a positive dielectric anisotropy and a phase sequence nematic$\rightarrow S_A \rightarrow S_C$. Confining such a system, being in the nematic phase, between two concentric glass cylinders, and applying a suitable electric field across these, a configuration with the nematic director pointing radially outwards from the cylinder axis will be formed. Lowering the temperature of the system below that of the nematic–S_A transition temperature, we expect a configuration consisting of cylindrical smectic layers to be formed. Such a configuration should also be geometrically stable when the electric field is released. Lowering the temperature further, taking the system into the S_C phase, the cylindrical configuration should remain. In this way we expect it to be possible to prepare a well defined S_C system consisting of concentric cylindrical layers with the common axis coinciding with that of the two glass cylinders. The ground state configuration of the c-director for such a system will be discussed in the next section. With this discussion in mind, the purpose of this paper is to discuss the possible formation of walls within the smectic layers when a magnetic field, making an angle with the cylinder axis, is applied over the sample.

2. The smectic C elastic layer energy

In this section we discuss the configuration of the c-director in a cylindrical smectic layer and show how the ground state configuration of the system depends upon the signs of some relevant elastic constants. To facilitate such a discussion we introduce the coordinates and notation according to figure 2. A cylindrical polar coordinate system (r, α, z), the z axis coinciding with the axis of the smectic layers, is introduced. The distance radially outwards is thus measured by r, while α is the polar angle. The basis vectors of this coordinate system are such that $\hat{\mathbf{r}}$ will coincide with the smectic layer normal while the smectic layers are parallel to the αz surface, $\hat{\boldsymbol{\alpha}}$ always pointing in the direction in which the layers are bending. The angle ϕ which is used to describe the c-director is defined as the angle between the z axis and the c-director, taking ϕ positive as is indicated in figure 2. Thus $\phi = 0$ corresponds to the state for which the c-director is parallel to the axis of the smectic cylinders and is pointing upwards in the way the figure is drawn. With these assumptions we can write down the following ansatz for the director $\mathbf{n}$ and the c-director expressed in cylindrical coordinates

$$\mathbf{n} = \hat{\mathbf{r}} \cos\theta + \hat{\boldsymbol{\alpha}} \sin\theta \sin\phi + \hat{\mathbf{z}} \sin\theta \cos\phi, \tag{2}$$

$$\mathbf{c} = \hat{\boldsymbol{\alpha}} \sin\phi + \hat{\mathbf{z}} \cos\phi. \tag{3}$$

The ground state of the system we study is one for which ϕ is constant throughout space. Even in this case there is, however, some elastic energy built into the system due to the bending of the smectic layers. The corresponding free energy density can be calculated [4] as

$$w_{\text{layer}}(\phi) = \frac{1}{2R^2}\left[A_{12} \sin^4\phi + A_{21} \cos^4\phi - 2A_{11} \sin^2\phi \cos^2\phi\right], \tag{4}$$

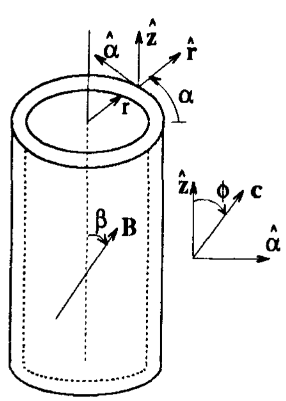

Figure 2. Definition of the coordinates used to describe the cylindrical smectic layers. The coordinate system used is a cylindrical one for which the z axis coincides with the axis of the smectic cylinders. The r direction is everywhere parallel to the smectic layer normal, and the smectic layers are parallel to the αz surface. The angle ϕ measures the rotation of the c-director with respect to the z axis, taking ϕ positive as is indicated in the right hand part of the figure.

where R is the radius of the layer we study and the A_{ij} constants are the three elastic constants associated with the bending of the smectic layers. From stability and symmetry reasons [4] these three constants must fulfil certain conditions; expanding the constants in powers of θ we can write

$$A_{12} = K + \bar{A}_{12}\theta^2, \tag{5a}$$

$$A_{21} = K + \bar{A}_{21}\theta^2, \tag{5b}$$

$$A_{11} = -K + \bar{A}_{11}\theta^2, \tag{5c}$$

where K and $\bar{A}_{ij}$ can be assumed to be only weakly temperature dependent. Furthermore, the following inequalities must be valid [4]:

$$K > 0, \tag{6a}$$

$$\bar{A}_{12} + \bar{A}_{21} + 2\bar{A}_{11} > 0. \tag{6b}$$

By the use of equations (5) the layer energy (4) can, apart from an irrelevant constant contribution, be written as

$$w_{\text{layer}}(\phi) = \frac{1}{2R^2}\,[(\bar{A}_{12} + \bar{A}_{11})\sin^4\phi + (\bar{A}_{21} + \bar{A}_{11})\cos^4\phi]\theta^2. \tag{7}$$

The form of this energy implies that the stable configuration ϕ_0 of the c-director will depend on the signs of the $\bar{A}_{ij}$ constants and we have therefore, to distinguish between three cases (the case $\bar{A}_{12} + \bar{A}_{11} < 0$ and $\bar{A}_{21} + \bar{A}_{11} < 0$ is excluded due to the inequality (6 b)). By minimizing equation (7) with respect to ϕ we can find the stable configuration ϕ_0 in each case

Case 1 $\bar{A}_{12} + \bar{A}_{11} > 0, \bar{A}_{21} + \bar{A}_{11} > 0 \Rightarrow \tan^2\phi_0 = (\bar{A}_{21} + \bar{A}_{11})/(\bar{A}_{12} + \bar{A}_{11}),$ (8 a)

Case 2 $\bar{A}_{12} + \bar{A}_{11} > 0, \bar{A}_{21} + \bar{A}_{11} < 0 \Rightarrow \quad \phi_0 = 0, \pi,$ (8 b)

Case 3 $\bar{A}_{12} + \bar{A}_{11} < 0, \bar{A}_{21} + \bar{A}_{11} > 0 \Rightarrow \quad \phi_0 = \pi/2, 3\pi/2,$ (8 c)

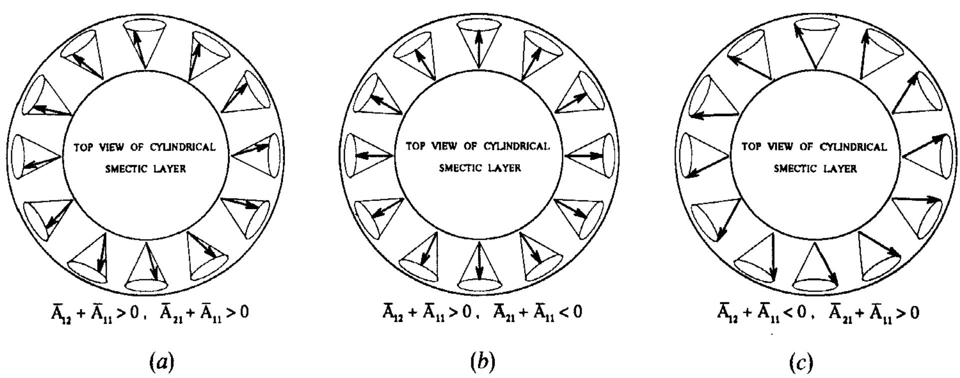

Figure 3. Top view of the smectic cylinder showing the stable director configuration as it depends on the signs of the elastic constants $\bar{A}_{12} + \bar{A}_{11}$ and $\bar{A}_{21} + \bar{A}_{11}$. The arrows in the figure represent the location of the director on the smectic cone. In (a) the c-director is pointing at an oblique direction with respect to the cylinder axis. In (b), on the other hand, the c-director is pointing parallel to the cylinder axis while in (c) the c-director is perpendicular to this axis.

where we also note that due to the symmetry of the system the repeated solutions in each of equations (8) represent the same physical state. Thus an examination of the c-director configuration in the proposed geometry will enable us to determine the signs of the constants $\bar{A}_{12}+\bar{A}_{11}$ and $\bar{A}_{21}+\bar{A}_{11}$ experimentally. In figure 3 we show the top view of the cylindrical layers and the corresponding equilibrium configuration of the director for the three different cases. As no experimental information regarding the $\bar{A}_{ij}$ constants is yet available, we cannot predict which of the three cases just discussed will be exhibited by the system. Our assumption is that all three cases will be feasible in reality.

3. The magnetic free energy density and the generalized magnetic torque

We now study the effect of applying a tilted magnetic field $\mathbf{B}=B\hat{\mathbf{B}}$ over a cylindrical smectic layer. We assume that the magnetic field makes an angle β to the cylinder axis and, as indicated in figure 2, the magnetic field is confined within the plane defined by the z axis and the direction $\alpha=0$. The unit vector $\hat{\mathbf{B}}$, defining the direction of the field, can be expressed by the cylindrical polar coordinate system as

$$\hat{\mathbf{B}}=\hat{\mathbf{r}}\sin\beta\cos\alpha-\hat{\boldsymbol{\alpha}}\sin\beta\sin\alpha+\hat{\mathbf{z}}\cos\beta. \tag{9}$$

The magnetic free energy density g_{m} can be expressed as [6]

$$g_{\mathrm{m}}=-\delta(\mathbf{n}\cdot\hat{\mathbf{B}})^2, \tag{10}$$

where δ is the coupling constant of the magnetic field

$$\delta=\tfrac{1}{2}\mu_0^{-1}\chi_{\mathrm{a}}B^2, \tag{11}$$

μ_0 being the permeability of free space and χ_{a} the magnetic anisotropy of the liquid crystal. From equations (2), (9) and (10) we calculate the magnetic free energy density of the system to be

$$g_{\mathrm{m}}=-\delta[\cos\theta\cos\alpha\sin\beta-\sin\theta\sin\alpha\sin\beta\sin\phi+\sin\theta\cos\beta\cos\phi]^2. \tag{12}$$

From this energy we can derive the expression of the generalized magnetic torque $\Gamma_{\mathrm{m}}=-\mathrm{d}g_{\mathrm{m}}/\mathrm{d}\phi$ acting on the director due to the presence of the field as

$$\begin{aligned}\Gamma_{\mathrm{m}}=\delta[&\sin^2\theta\sin^2\alpha\sin^2\beta\sin 2\phi-\sin^2\theta\cos^2\beta\sin 2\phi\\ &-2\sin\theta\cos\theta\sin\alpha\cos\alpha\sin^2\beta\cos\phi\\ &-2\sin\theta\cos\theta\cos\alpha\sin\beta\cos\beta\sin\phi-2\sin^2\theta\sin\alpha\sin\beta\cos\beta\cos 2\phi].\end{aligned} \tag{13}$$

Disregarding the influence of elastic forces, the angles ϕ for which Γ_{m} vanishes represent the equilibrium directions of the c-director in the magnetic field.

4. Formation of walls

In this section we show how the application of a magnetic field making an angle β to the axis of the cylindrical smectic layers in accord with figure 2 must, for a strong enough field, lead to the formation of a number of walls running parallel to the cylinder axis. We discuss the case of positive magnetic anisotropy ($\delta>0$) leaving the obvious changes imposed on the analysis in the case $\delta<0$ to the reader. Generally, in the presence of the magnetic field, the molecules in the case we study want to rotate in such a way that the director becomes parallel to the field. For the $\mathrm{S_C}$ liquid crystal the motion of the director is, however, restricted to align on the smectic cone, and the effect of the field can only be to minimize the angle between the director and the field. Depending on

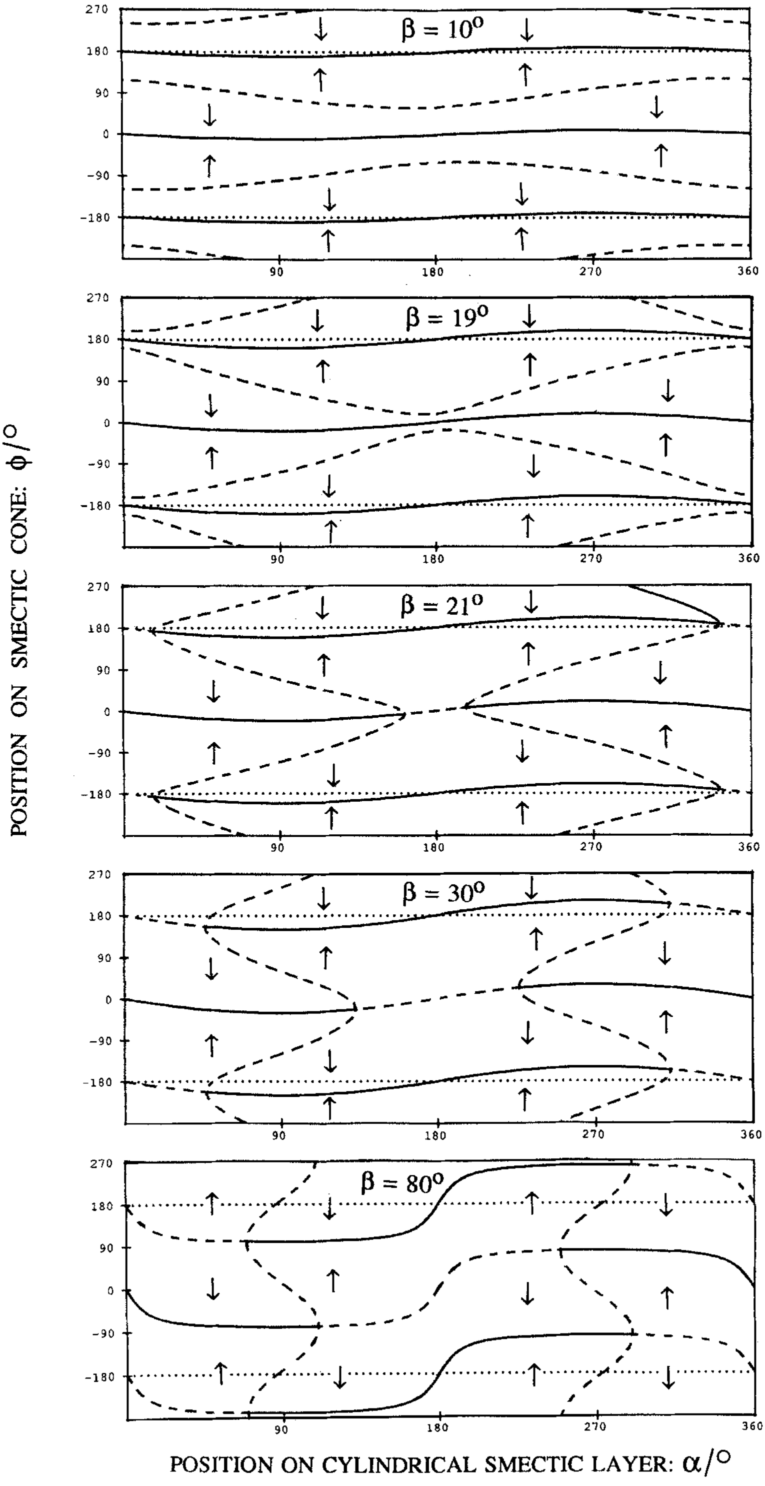

Figure 4. Torque maps for a tilt angle θ of $20°$ calculated for five different values of the inclination β of the magnetic field with respect to the axis of the smectic cylinders. Notice the qualitative topological difference between the torque maps depending on whether β is smaller or larger than θ. ——, Indicates stable equilibrium, – – –, unstable equilibrium and ↑ ↓ denote the sense of rotation for the c-director.

which point of the cylinder we are investigating, this will be achieved by a clockwise or counterclockwise rotation. Which particular case occurs depends on the sign of the generalized magnetic torque Γ_m given by equation (13).

From equation (13) we can construct torque maps from which we can deduce the behaviour of the system in the presence of the field. In figure 4 we show such torque maps calculated for a tilt angle of 20° for five different values of the inclination of the magnetic field, namely $\beta = 10°$, 19°, 21°, 30° and 80°. The graphs in the figure represent the function $\phi(\alpha)$ for which the magnetic torque vanishes. Thus the graphs show how the equilibrium positions (disregarding elasticity) of the c-director change as we travel around the smectic cylinder. If a perturbation of the c-director from an equilibrium position creates a torque tending to bring it back to the original position, the equilibrium is stable and is plotted as a full line, while unstable equilibrium positions are plotted as dashed lines. For each point on the cylinder, i.e. for each value of α, we have drawn vertical arrows in the figure showing in which direction the magnetic field forces the c-director to rotate for a given value of ϕ. We have extended the ϕ-axis to cover the interval $\phi \in [-270°, 270°]$. However, we have drawn two horizontal dotted lines corresponding to $\phi = \pm 180°$ in the figure; everything which falls outside these lines contains redundant information and is only included in order to make the interpretation of the figure more transparent. From the figure we see that the torque maps exhibit a qualitatively different topology depending on whether the inclination β of the magnetic field is larger or smaller than the tilt angle θ. In the case $\beta < \theta$ (for example, when $\beta = 10°$ and 19°) there are always two stable and two unstable equilibrium positions of the c-director irrespective of which point on the smectic cylinder we study. When, on the other hand, $\beta > \theta$ (for example, when $\beta = 21°$, 30° and 80°), we see that in two intervals around $\alpha = 0°$ and 180° there is one stable and one unstable equilibrium only, while in the two intervals around $\alpha = 90°$ and 270° there are two stable and two unstable positions of the c-director.

We are now in a position to understand how walls in many cases are inevitably formed in the system when the magnetic field is applied. In § 2 we showed that the stable c-director configuration of the system, in the absence of the field, is the one for which ϕ is constant all around the cylinder, adopting an equilibrium value ϕ_0 which is some angle between zero and 90° depending on the signs of the $\bar{A}_{ij}$ constants. If we draw the corresponding horizontal line in a specific torque map we will for each value of α (i.e. for each point on the cylindrical smectic layer) see in which direction the director will rotate due to the field. If the line $\phi = \phi_0$ intersects an unstable equilibrium curve, the sense of rotation will conflict on either side of the corresponding point. If the magnetic field is strong enough to overcome the elastic forces, a wall is formed at this point. If we have prepared our sample homogeneously in space, this wall extends in the z and r directions and has the shape of a plane.

We now show that, depending on the topology of the torque map, two different kinds of walls can be created. In figure 5 we show the situation for which $\beta < \theta$, assuming the case $\phi_0 = 90°$. In the upper part of the figure we show the corresponding torque map and the line $\phi = \phi_0$. This line intersects an unstable equilibrium curve twice and two walls are created, having twists of $\Delta\phi = 180°$ and $-180°$. In the middle part of the figure we show the resulting equilibrium values of ϕ as a function of α while the lower part shows how the c-director rotates on the smectic cone as we travel around the cylindrical smectic layer. In reality the walls will, of course, not be infinitely thin but will be smeared out by elasticity. We will however show, in the next section, that this smearing out under reasonable experimental conditions will only be of the order of a

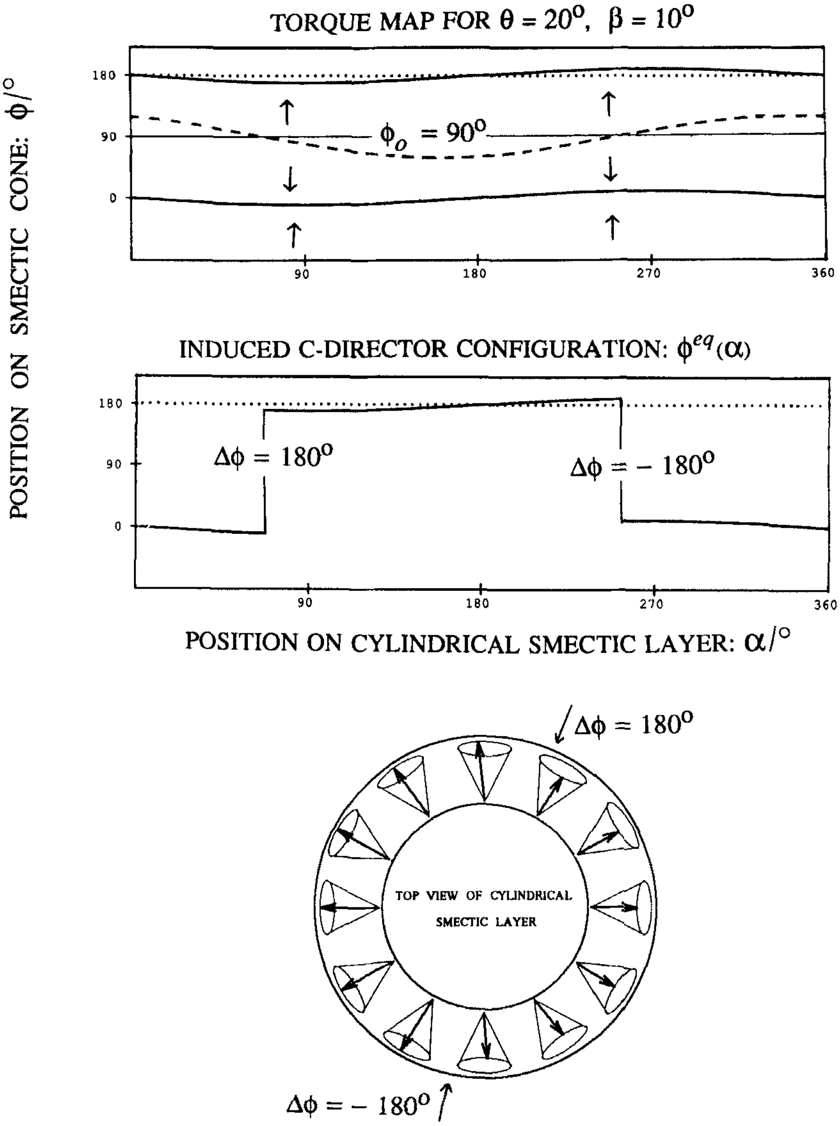

Figure 5. Wall formation in the case $\beta<\theta$. The upper part of the figure shows the relevant torque map in which a thin line indicates one possible c-director configuration in the absence of the magnetic field, $\phi_0=90°$. Each time this line intersects an unstable equilibrium curve, the sense of rotation for the c-director will be conflicting on either side of the corresponding point and thus a wall is created. In this particular case two walls are formed and the middle part of the figure shows how the angle ϕ between the c-director and the cylinder axis varies as a function of the position α on the cylindrical smectic layer. The lower part of the figure shows a top view of a smectic cylinder indicating how the c-director rotates on the smectic cone as we travel around this layer.

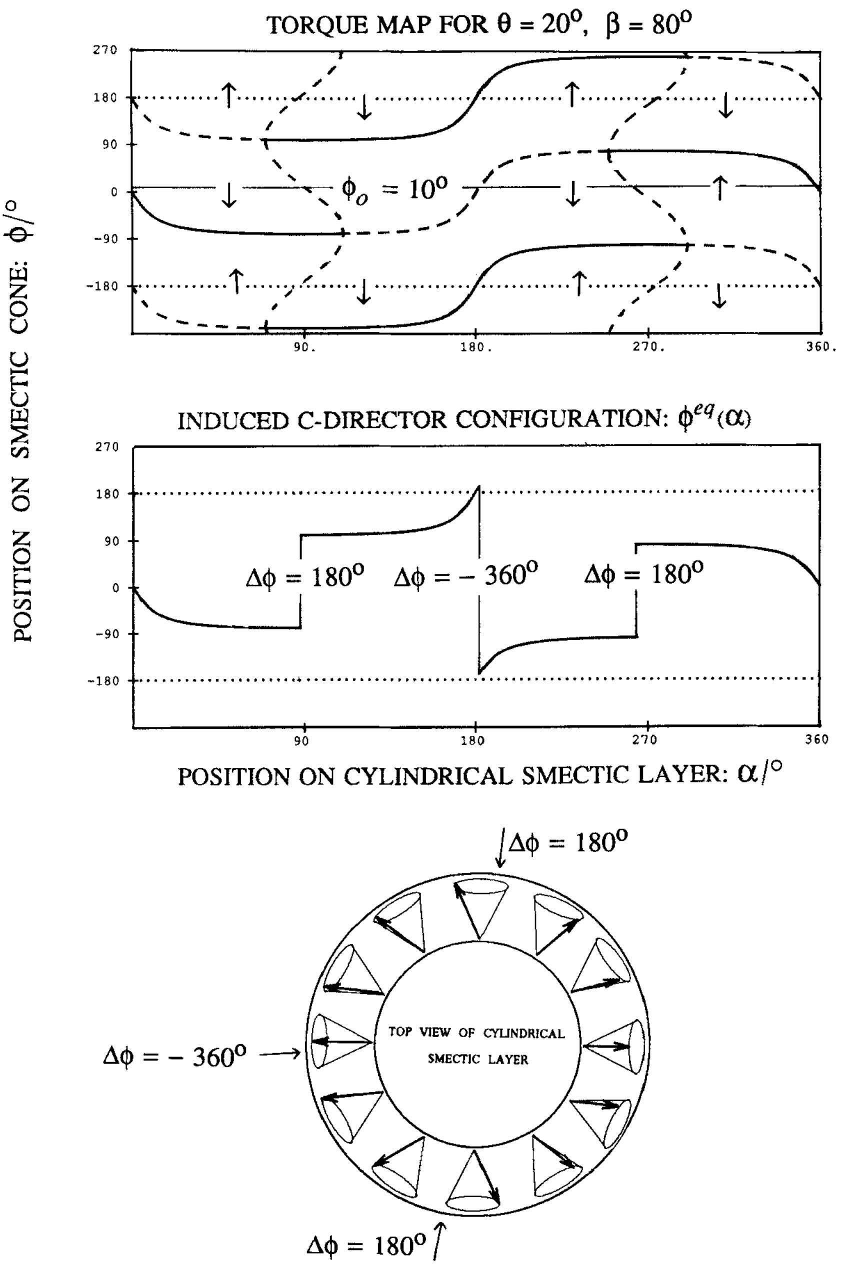

Figure 6. Wall formation in the case $\beta > \theta$. The upper part of the figure shows the relevant torque map in which a thin line indicates one possible c-director configuration in the absence of the magnetic field, $\phi_0 = 10°$. Each time this line intersects an unstable equilibrium curve, the sense of rotation for the c-director will be conflicting on either side of the corresponding point and thus a wall is created. In this particular case three walls are formed and the middle part of the figure shows how the angle ϕ between the c-director and the cylinder axis is varying as a function of the position α on the cylindrical smectic layer. The lower part of the figure shows a top view of a smectic cylinder indicating how the c-director rotates on the smectic cone as we travel around this layer.

fraction of a degree. The situation for which $\beta > \theta$ is shown in figure 6. Here there are two different possible cases depending on the value of ϕ_0. If ϕ_0 is large enough, there are again two intersections between the line $\phi = \phi_0$ and unstable equilibrium curves, and two 180° walls are created in the same manner as in the case discussed previously. If, however, ϕ_0 is small enough we find three intersections. This is the case depicted in figure 6 where we have chosen $\phi_0 = 10°$; here we see that three walls are created, two with $\Delta\phi = 180°$ and one with $\Delta\phi = -360°$.

From figures 5 and 6 we conclude that, depending on the values of θ, β and ϕ_0, two different types of wall formation can occur where, provided the magnetic field is strong enough, either two or three walls are formed in the cylindrical smectic layers. It is also clear that in some cases no walls are formed, but the magnetic field merely induces a structure with a smoothly varying $\phi^{eq}(\alpha)$. Which case will occur depends on how many intersections there are between the line $\phi = \phi_0$ and the unstable equilibrium curves.

5. Estimation of the wall thickness

We now proceed to estimate the typical thickness of the walls discussed in the previous section. The equations governing the elastic behaviour of the c-director in a cylindrical smectic layer has been derived by us elsewhere (see equation (27) in [4]). In the case studied in the present work we have to add the term $R^2\Gamma_m/\theta^2$ to this equation, R being the radius of the layer we study and Γ_m the magnetic generalized torque given by equation (13). Generally we now have to introduce two additional elastic constants B_1 and B_2 into the equations. As very little experimental information is as yet available regarding their magnitudes, we introduce the approximation $B^{el} = B_1 = B_2$ in order to simplify the present analysis. Assuming the system to be relaxed in the r and z directions, a general elastic deformation of the c-director within a smectic layer is now given by

$$B^{el}\frac{d^2\phi}{d\alpha^2} + 2[(\bar{A}_{21} + \bar{A}_{11})\cos^2\phi - (\bar{A}_{12} + \bar{A}_{11})\sin^2\phi]\theta^2 \sin\phi\cos\phi + R^2\Gamma_m = 0. \quad (14)$$

This equation can be considered as a balance of torque equation in which three different torques are at equilibrium with each other. The first term represents the normal deformational torque, appearing as soon as the c-director is non-uniform ($d\phi/d\alpha \neq 0$) with respect to the smectic layers. The second term will exist even if ϕ is constant, unless ϕ equals one of the equilibrium values discussed in § 2, and has its origin in the fact that the smectic layers are bending. Finally, the third term is the torque due to the application of the magnetic field.

To use equation (14) to calculate the thickness of the walls created by the magnetic field we would need to perform a numerical integration of this equation. The wall thickness will also depend upon where on the cylinder it appears, because in different positions the magnetic field is oriented differently with respect to the smectic layer. We will, however, show that we can use equation (14) to obtain a crude estimate of the wall thickness. As no experimental information exists regarding the $\bar{A}_{ij}$ constants, and as the corresponding term only introduces a quantitative change in the features of equation (14), we will neglect the influence from this term in our estimate. Introducing equation (13) into equation (14), the latter can, disregarding the term containing the $\bar{A}_{ij}$ constants, be written as

$$\frac{B^{el}}{R^2\delta}\frac{d^2\phi}{d\alpha^2} = f_1(\theta, \alpha, \beta, \phi), \quad (15)$$

where f_1 is a function obtained from equation (13). By introducing an angle γ_w according to

$$\gamma_w^2 = \frac{B^{el}}{R^2 \delta}, \tag{16}$$

and studying equation (15) close to one of its equilibrium solutions ϕ^{eq}, we can expand the latter equation to read

$$\gamma_w^2 \frac{d^2\phi}{d\alpha^2} \approx (\phi - \phi^{eq}) f_2(\theta, \alpha, \beta), \tag{17}$$

where f_2 is some other general function. If we assume that the magnetic field is strong enough to confine the walls to a narrow interval $\Delta\alpha$ on the cylinder, the function f_2 can be treated as a constant, the value of which depends on the tilt angle θ, the inclination of the magnetic field β and where on the cylinder the wall will be formed. The value of f_2 seems, when putting $\theta = 20°$, generally to fall within the interval $[0{\cdot}1, 10]$ and for convenience we now assume f_2 to equal unity. In such a case equation (17) describes a relaxation behaviour of ϕ with the correlation angle γ_w. From equations (11) and (16) we thus estimate the thickness of the walls (neglecting any numerical constants of order unity) to be

$$\gamma_w = \frac{1}{RB}\sqrt{\left(\frac{B^{el}\mu_0}{\chi_a}\right)}, \tag{18}$$

where we have expressed the thickness in terms of the polar angle of the cylinder over which the wall extends. A typical value of the magnetic anisotropy can be taken to be [6] $\chi_a \approx 10^{-6}$, while one of the few experimental estimations of B^{el} existing today reveals [7] $B^{el} \approx 10^{-11}$ N for a S_C system where the tilt angle is close to 20°. Assuming a magnetic field of 1 T to be used, putting the radius of the smectic cylinders equal to 5 mm, we now obtain a numerical estimate of γ_w ($\mu_0 = 4\pi \times 10^{-7}$ Vs/Am) to be $\gamma_w \approx 7 \times 10^{-4}$ rad $\approx 0{\cdot}04°$. The estimate of γ_w presented here is of course rather crude, but we have nevertheless shown that under reasonable experimental conditions the extension of the walls in the α direction is of the order of a small fraction of a degree. Thus the walls in most cases should be sharp and clearly observable.

6. Discussion

Studies of the elastic properties of smectic layers are currently very scarce. This is because the geometrical constraint $\nabla \times \mathbf{a} = 0$ in most cases stabilizes the structure of the smectic layering. It follows that an external disturbance imposed on the system will rarely perturb the smectic layer structure and so will only have an influence on the c-director. We have, however, shown previously [4], and in this work, that by a suitable geometrical set-up it is possible to gain some information of the S_C elastic layer energy, even if the layers themselves stay intact. This is achieved by using the interplay between layer deformations and c-director rotations in a system with a cylindrical arrangement of the smectic layers. By studying the c-director rotations in such a system we can gain some information regarding the elastic properties of the layers in a S_C liquid crystal. We demonstrated in § 2 (cf. figure 3) that the ground state of the c-director in a cylindrical smectic layer will belong to one of three cases, depending on the signs of the crucial elastic constants $\bar{A}_{21} + \bar{A}_{11}$ and $\bar{A}_{12} + \bar{A}_{11}$. If these two combinations of elastic constants are of opposite sign, the c-director will either point parallel to the cylinder axis ($\bar{A}_{12} + \bar{A}_{11} > 0$) or in the same direction as the layers are bending ($\bar{A}_{21} + \bar{A}_{11} > 0$). If both of these combinations of constants are positive, the c-director will adopt some intermediate angle, the value of which is given by equation (8 a). Thus the signs of the constants $\bar{A}_{12}+\bar{A}_{11}$ and $\bar{A}_{21}+\bar{A}_{11}$, and possibly also their ratio, can be determined by studying a relaxed cylindrical S_C layer.

Applying an oblique magnetic field over the system will, under some circumstances, create walls within the cylindrical smectic layers. In order to analyse this situation we constructed the torque maps shown in figure 4. In these maps it was shown how the equilibrium position of the c-director in the presence of the magnetic field is shifted as we travel around the smectic layer. Thus we can show by the construction in figures 5 and 6, how walls in some cases inevitably must be formed if a magnetic field is applied over the system at an angle to the axis of the smectic cylinder. Depending on the circumstances, two ($\Delta\phi=\pm 180°$) or three ($\Delta\phi=180°$, $\Delta\phi=-360°$, $\Delta\phi=180°$) walls will be observed. The discussion in § 3 and 4 assumed the magnetic anisotropy of the sample to be positive ($\delta>0$). In the case $\delta<0$ the only difference in the analysis of the torque maps is that the stable and unstable lines will change place and the arrows indicating the sense of rotation of the c-director will be reversed. The qualitative nature of the wall pattern in this case will thus be unchanged; however, the walls will appear in different positions of the cylinder.

Most effects exhibited by a liquid-crystalline system in the presence of a magnetic field are reproducible if instead an electric field is applied across the sample. However, in the case of the electric field the analysis of the behaviour of the system becomes more involved, because an inhomogeneous director configuration causes the electric field also to become inhomogeneous. Furthermore, the recently reported [8] appearance of dielectric biaxiality of S_C systems will make the response of the system to electric fields even more complex. As can be understood from geometrical considerations, the qualitative nature of the wall formation will still be the same in the case of electric fields. However, we cannot exclude the possibility that the dielectric biaxiality, if it is large enough, can give the torque maps of figures 4–6 a more complex appearance, and thus the system in this case will accordingly change its response to the electric field.

In § 5 we estimated the typical thickness of the walls to be of the order of a few per cent of a degree. Thus we are assured that the walls will be distinct and clearly observable. In the derivation of the coherence angle γ_w we neglected the influence of the $\bar{A}_{ij}$ constants in equation (14). Including the corresponding term into the analysis will lead to a renormalization of the denominator in the expression of γ_w^2, $R^2\delta \rightarrow R^2\delta + f(\bar{A}_{ij})$, where $f(\bar{A}_{ij})$ is some general function of the $\bar{A}_{ij}$ constants, depending on the values of θ and β. If the wall thickness γ_w is measured, and if we assume that the constant B^{el} has been determined by some independent method, this renormalization of $R^2\delta$ would then make it possible to estimate the specific combination of the $\bar{A}_{ij}$ constants which is contained in $f(\bar{A}_{ij})$ in the particular case we have studied.

The basis for the problem discussed in this work is the possibility of preparing a S_C liquid-crystalline system for which the smectic layers form a set of well defined concentric cylinders. We are well aware of the fact that the experimentalist might face some unsolvable problems when trying to prepare such a system. However, as the walls which the system developed in the presence of the field are expected to have a thickness of a fraction of a degree, there is the possibility of circumventing these potential problems by instead preparing a sample in which the smectic layers only extend over part of a cylinder. This could be achieved by using glass plates which are slightly curved in a proper way. Although we will not be able to study the full set of walls in such a sample at the same time, we may by rotating the system in the magnetic field, still be able to study the walls one at a time.

References

[1] DE GENNES, P. G., 1975, *The Physics of Liquid Crystals* (Clarendon Press).
[2] CARLSSON, T., and DAHL, I., 1983, *Molec. Crystals liq. Crystals*, **95**, 373.
[3] STEWART, I. W., NAKAGAWA, M., and LESLIE, F. M. (to be published).
[4] CARLSSON, T., STEWART, I. W., and LESLIE, F. M., 1991, *Liq. Crystals*, **9**, 661.
[5] GRAY, G. W., and GOODBY, J. W. G., 1984, *Smectic Liquid Crystals* (Leonard Hill).
[6] DE JEU, W. H., 1980, *Physical Properties of Liquid Crystalline Materials* (Gordon & Breach).
[7] PELZL, G., SCHILLER, P., and DEMUS, D., 1987, *Liq. Crystals*, **2**, 131.
[8] JONES, J. C., RAYNES, E. P., TOWLER, M. J., and SAMBLES, J. R., 1990, *13th International Liquid Crystal Conference*, Vancouver.